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Convergence Theorem for Junck-Mann Iteration in Banach Space

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ABSTRACT:

In this paper, we establish the week convergence the sequence of Mann iteration of quasi-nonexpansive maps in the framework of uniformly convex Banach space. The results obtained generalize some well known existing results.

M.S.C. – 47H10, 47H09, 47H20, 54E25

Keywords – Mann iteration, I – quasi- nonexpansive map

1. INTRODUCTION:

Compatible maps and generalization of commuting maps are characterized. In term of coincidence points and common fixed point theorem for commuting maps. Gerald Junck introduced this type of iteration in 1988. We use nonexpansive and quasi- nonexpansive mapping in the Junck iteration. We remark that the class of quasi- nonexpansive maps properly includes the class of nonexpansive maps with $F(T) \neq \Phi$ [1] . Gosh and Debnath [2] studied the convergence of iterates of the family of nonexpansive mapping in a uniformly convex Banach space. Rhoades and Temir [3] established the weak convergence of the sequence of the Mann iterates to a common fixed point of T and I by considering the map T to be I- nonexpansive.

Recently Kizilton c and Ozdemir established the week convergence of the sequence of modified Ishikawa iterates to a common fixed point of T and I. kuman, Kumethong and Jewwaiworn [4] also established the week convergence for an I- nonexpansive mapping in Banach space. Our aim is to establish the weak convergence of the sequence of Mann iteration to a common fixed point of two maps T and I.

The Mann iteration scheme [5], for n= 0, 1, 2 …………………. and $\alpha_2 c[01]$ is defined as

$$X_{n+1} = (1-\alpha_n)\, x_n + \alpha_n T x_n \qquad (1)$$

Further these iterative schemes are developed by taking two mapping S, T : $Y \longrightarrow x$ where $T(Y) \subseteq S(Y)$ and $x_0 \in Y$. Singh et at [6] discuss the following iterative procedure.

$$Sx_{n+1} = f(T, x_n),\ n = 0, 1, \ldots \qquad (2)$$

It is called Junek iterative procedures [7]. It $f(T, x_n)$ in [8,9] is replaced by Tx_n

$$(1-\alpha_n)\, Sx_{n+1} + \alpha_n Tx_n \qquad (3),$$

it becomes Junck Picard and Junck Mann iteration.

2. PRELIMINARIES:

Let K be a closed convex banded subset of a uniformly concave Branch space (X,II.II) and T be a self mapping of X. T is nonexpansive on K if for all x, y ∈ K we have.

$$\text{II}Tx-Ty\text{II} \leq \text{II}x-y\text{II} \qquad (4)$$

A point of f ∈ K is a fixed point of T if Tf =f. We denote the set of the fixed points of T by f (T), where
f (T) ={f ∈ K : T **f** =f}

A map T satisfying

$$\text{II } Tx-f \text{ II} \leq \text{II } x-f \text{ II} \qquad (5)$$

X ∈ K and f ∈ F (T), is called a quasi- nonexpansive mapping.

Definition 2.1- T is called I- nonexpansive map on K if IITx-TyII ≤ IITx-Ty,for all x, y, ∈ K T is called I- quasi nonexpansive map on K if II Tx-f II ≤ II Tx-f II for all x, y ∈K is a common fixed point of I and T if x=Ix==Tx=Sx.

3. MAIN RESULT:

Theorem – Let K be a closed, convex and bounded subset of a uniformly convex banach space, and let T and I non self mapping of K with T be an I—quasi-nonexpansive and I a nonexpansive on K. Then x_0 ∈ K the sequence $\{x_n\}$ of iterates defined by (3) converges weakly to common fixed point of F (T) ∩F(S).

Proof- If F(T) ∩F(S) ≠ Ø we will assume, and F(T)∩ F(S) is not a singleton.

Since,

$$\begin{aligned} \text{II } Sx_{x+1} - f \text{ II} &= \text{II } (I-\alpha_n)(Sx_n+\alpha n\, Tx_n) - f \text{ II} \\ &= \text{II } (I-\alpha_n)(Sx_n-f)+ \alpha_n (Tx_n-f) \text{ II} \\ &= \text{II } (I- \alpha_n)(Ix_n-f) + \alpha_n (Ix_n-f) \text{ II} \\ &= \text{II } (I- \alpha_n)(x_n-f) + \alpha_n (x_n-f) \text{ II} \\ &= (I- \alpha_n) \text{ II } x_n-f \text{ II} + \alpha_n (1x_n-f) \text{ II} \end{aligned}$$

I $\quad$ I $Sn_{n+1} - f$ II = II $x_n - f$ II

$\alpha_n \neq o$, { IIxn-f II } is a non increasing sequence.

Then

lim II $x_n - f$ II exist

[6-] when Y=X and S = id = I is the identity operator on X.

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